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THE DISTRIBUTION OF LOADS ON RIVETS CONNECTING A PLATE
TO A BEAM UNDER TRANSVERSE LOADS

By F. Vogt

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THE DISTRIBUTION OF LOADS ON RIVETS CONNECTING A PLATE
TO A BEAM UNDER TRANSVERSE LOADS¹

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SUMMARY

This report gives a theoretical discussion of the distribution of loads on rivets connecting a plate to a beam under transverse loads. Two methods of solution are given which are applicable to loads up to the limit of proportionality; in the first the rivets are treated as discrete members, and in the second they are replaced by a continuous system of jointing. A method of solution is also given which is applicable to the case when nonlinear deformations occur in the rivets and the plate, but not in the beam.

The methods are illustrated by numerical examples, and these show that the loads carried by the rivets and the plate are less than the values given by classical theory, which does not take into account the slip of the rivets, even below the limit of proportionality. This difference is considerably accentuated when nonlinear deformations occur in the structure and the beam then carries the greater portion of the bending moment. If the material of the beam has a higher proportional limit and a higher ultimate strength than the material of the plate, there is thus a transfer of load from weaker to stronger material, and this is to the advantage of the structure.

The methods given are of simple application and are recommended for use in the design of light-alloy structures when the design load is likely to be above the proportional limit.

INTRODUCTION

This report contains a theoretical discussion of the distribution of loads on rivets connecting a plate to a beam under transverse loads,

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and the analysis is developed on similar lines to those used by the present author in determining the distribution of loads in riveted joints (reference 1). No allowance is made for any inter-rivet buckling of the plate.

Two methods of solution are given which are applicable to loads up to the limit of proportionality; in the first the rivets are treated as discrete members, and in the second they are replaced by a continuous system of jointing. A method of solution is also given which is applicable to the case when nonlinear deformations occur in the rivets and the plate but not in the beam. A number of numerical examples is given to illustrate the methods of analysis and conclusions are drawn from the results obtained.

If the width of the plate is large in comparison with the length of the structure, only part of the plate will be effective as a load carrying member. Although this problem is not entered into here, it is of considerable importance in the design of stressed skin and reinforced concrete structures, and reference may be made to the known approximate solutions given in reference 1 on the subject.

SOLUTION APPLICABLE TO LOADS UP TO THE PROPORTIONAL LIMIT

First Method - Rivets Treated As Discrete Members

The structure is assumed to be arbitrarily loaded as shown in figure 1 and the moment diagram is also assumed to be known. Let

A_1 area of the beam

A_2 area of the plate

I_1 moment of inertia of the beam about an axis through the center of mass of the beam

I_2 moment of inertia of the plate about an axis through the center of mass of the beam (in practice I_2 is small and is of little importance in the analysis of the load distribution)

a distance between the centers of mass of the beam and the plate

$I = I_1 + I_2 + A_1 A_2 a^2 / (A_1 + A_2)$, moment of inertia of the whole section (i.e. the beam and the plate) about an axis through the center of mass of the whole section

The stiffness of the structure is not, however, known in terms of I because rivet slip occurs, and the angle of bending ϕ for a length l of the structure (as shown in fig. 2) is therefore not assumed to be known.

Let

$$\delta_1 = CP_1 \delta_0, \text{ slip at } i^{\text{th}} \text{ rivet under a load } P_i$$

and

$$\delta_{i+1} = CP_{i+1} \delta_0, \text{ slip at } (i+1)^{\text{th}} \text{ rivet under a load } P_{i+1},$$

where

$$\delta_0 = l/EA_0$$

is the extension of a section of area A_0 under unit load and C is a coefficient representing the stiffness of the rivet. The total axial load in the structure is zero and hence the total tensile load N_i in the plate is equal to the total compressive load in the beam. There is clearly no axial load in the plate to the left of the first rivet, and in general

$$N_i = P_1 + P_2 + \dots + P_i$$

The bending moment in the structure at a section midway between two rivets is denoted by M_i can then

$$M_i = E(I_1 + I_2)\phi_i/l + aN_i$$

Consideration of the extension at the common surface of the plate and the beam gives the following relation between the rivet slip at two successive rivets

$$N_i l/EA_2 - t_2 \phi_i/2 + \delta_i = N_i l/EA_1 + (a - t_2/2)\phi_i + \delta_{i+1}$$

that is

$$\begin{aligned} \phi_i &= \left\{ \delta_i - \delta_{i+1} + N_i(l/A_1 + l/A_2)/E \right\} / a \\ &= (l/EA) \left\{ (C/A) (P_i - P_{i+1}) + (1/A_1 + 1/A_2) \right\} \end{aligned}$$

and on substitution into the above equation for M_1 it is found that

$$M_1 = (I_1 + I_2) \left\{ (C/A) (P_1 - P_{i+1}) + N_1 (1/A_1 + 1/A_2) \right\} / a + a N_1$$

that is

$$P_{i+1} = P_1 + (A/C) N_1 \left\{ 1/A_1 + 1/A_2 + a^2 / (I_1 + I_2) \right\} - a M_1 / (I_1 + I_2).$$

If there are variations not only in the cross-sectional area but also in the pitch and the stiffness of the rivets along the beam, the corresponding equation may be found in the following way. Let l_0 and A_0 be the rivet pitch and cross-sectional area at some standard section and

$$\delta_0 = l_0 / EA_0$$

The slip at the i^{th} rivet is now written as

$$\delta_i = C_i P_i \delta_0$$

where C_i is the rivet stiffness and it is then found that

$$\begin{aligned} M_1 &= E(I_1 + I_2) \phi_i / l_i + a N_1 \\ &= \left\{ P_1 - P_{i+1} + N_1 l_i (1/A_1 + 1/A_2) / E \right\} \end{aligned}$$

from which

$$P_{i+1} = (C_i / C_{i+1}) P_i + (l_i A_0 / l_0 C_{i+1}) \left[N_1 \left\{ 1/A_1 + 1/A_2 + a^2 / (I_1 + I_2) \right\} - a M_1 / (I_1 + I_2) \right]$$

where the length between the i^{th} and $(i+1)^{\text{th}}$ rivets is l_i and A_1 , A_2 , I_1 , and I_2 are implicitly understood to have the suffix i associated with them.

If the moment diagram for the structure is known, the load on each rivet may be expressed in terms of the load P_1 on the first rivet and this load is determined from the equation

$$N_H = 0 = P_1 + P_2 + \dots + P_n$$

The analysis is of course simplified when there is symmetry about the center line of the structure

When the rivets are absolutely rigid, that is $C = 0$,

$$N_1 = M_1/k$$

where

$$k = \left\{ I_1 + I_2 + A_1 A_2 a^2 / (A_1 + A_2) \right\} / \left\{ A_1 A_2 a / (A_1 + A_2) \right\}$$

and this result corresponds to the classical theory where the rivet loads are obtained from the increase in M between successive rivets. This load distribution is, however, considerably altered by the slip of the rivets.

Example 1

Consider a simply supported beam under a total load Q uniformly distributed along the span L , as shown in figure 3, thus giving a maximum bending moment M equal to $QL/8$. The span and rivet pitch are taken to be 20 inches and 2 inches, respectively, and the other dimensions are shown in figure 4. It is found that

$$A_1 = 0.360 \text{ inch}^2$$

$$A_0 = A_2 = 0.116 \text{ inch}^2$$

$$I_1 = 0.01080 \text{ inch}^4$$

$$I_2 = 0.00013 \text{ inch}^4$$

and

$$a = 0.358 \text{ inch}$$

Assume $C = 3.84$ (this value of the rivet stiffness is chosen because it gives simple numbers for the final coefficients) corresponding to a rivet slip of

$$\delta = (P/Ea) f = CP(l/EA_0), \text{ that is, } C = f(A_0/l\delta)$$

where d is the rivet diameter and $f = 10.3$. Then

$$(A_0/C) \left\{ 1/A_1 + 1/A_2 + a^2/(I_1 + I_2) \right\} = 0.7 \quad \text{and} \quad A_0 a/C(I_1 + I_2) = 1.0.$$

The bending moments at sections midway between the rivets are

$$M_1 = M_9 = 0.9Q$$

$$M_2 = M_8 = 1.6Q$$

$$M_3 = M_7 = 2.1Q$$

$$M_4 = M_6 = 2.4Q$$

and

$$M_5 = 2.5Q$$

The equations to be solved are now

$$\begin{aligned} P_1 &= & &= 1.000 P_1 \\ P_2 &= P_1 + 0.7P_1 - M_1 & &= 1.700 P_1 - 0.900Q \\ P_3 &= P_2 + 0.7(P_1 + P_2) - M_2 & &= 3.590 P_1 - 3.130Q \\ P_4 &= P_3 + 0.7(P_1 + P_2 + P_3) - M_3 & &= 7.993 P_1 - 8.051Q \\ P_5 &= P_4 + 0.7(P_1 + P_2 + P_3 + P_4) - M_4 & &= 17.991 P_1 - 18.908Q \\ P_6 &= P_5 + 0.7(P_1 + P_2 + P_3 + P_4 + P_5) - M_5 & &= 40.583 P_1 - 43.100Q \end{aligned}$$

and from symmetry

$$P_6 = -P_5$$

that is,

$$40.583 P_1 - 43.100Q = -17.991 P_1 + 18.908Q$$

that is,

$$P_1 = 1.059Q$$

and it follows that

$$P_2 = 0.900Q$$

$$P_3 = 0.671Q$$

$$P_4 = 0.412Q$$

$$P_5 = 0.141Q$$

and

$$N_5 = P_1 + P_2 + \dots + P_5 = 3.183Q$$

The bending moments M_1, M_2, \dots, M_5 have been taken to be equal to the values midway between successive rivets, and if the average values of the bending moments between successive rivets are taken instead the following more accurate results are obtained,

$$P_1 = 1.052Q$$

$$P_2 = 0.897Q$$

$$P_3 = 0.670Q$$

$$P_4 = 0.412Q$$

$$P_5 = 0.141Q$$

and

$$N_5 = 3.172Q$$

The error introduced by the above approximation for the bending moments is therefore unimportant.

The results given by the classical theory (i.e., infinitely stiff rivets) are

$$P_1 = 1.286Q$$

$$P_2 = 1.000Q$$

$$P_3 = 0.715Q$$

$$P_4 = 0.429Q$$

$$P_5 = 0.143Q$$

and

$$N_5 = 3.573Q$$

and a comparison with the previous results shows that rivet slip has reduced the maximum rivet load by about 18 percent and the tensile load in the plate by about 11 percent.

Example 2

If the structure is extended beyond the supports with an additional rivet at each end as shown in figure 5, similar calculations give the results

$$P_0 = 0.469Q$$

$$P_1 = 0.798Q$$

$$P_2 = 0.784Q$$

$$P_3 = 0.620Q$$

$$P_4 = 0.390Q$$

$$P_5 = 0.133Q$$

and

$$N_5 = 3.194Q$$

that is, the additional rivet at each end of the structure reduces the maximum rivet load to only 62 percent of that found by classical theory and to 75 percent of the more accurate result.

Example 3

Suppose now, that instead of being simply supported, the structure is continuous over a large number of spans. The loading and dimensions of the structure are taken to be the same as before and the moment diagram is shown in figure 6. From symmetry

$$P_0 = -P_1$$

and

$$P_5 = -P_6$$

and if N_0 is the axial load in the plate at the supports

$$N_1 = N_0 + P_1$$

$$N_2 = N_1 + P_2, \text{ and so forth.}$$

Then with the same dimensions as in example 1,

$$P_{i+1} = P_i + 0.7 N_i - M_i$$

where the bending moments, taken to be the average values between successive rivets, are

$$M_0 = - 1.422Q$$

$$M_1 = - 0.767Q$$

$$M_2 = - 0.067Q$$

$$M_3 = + 0.433Q$$

$$M_4 = + 0.733Q$$

and

$$M_5 = + 0.833Q$$

The equation

$$P_1 = P_0 + 0.7 N_0 + 1.422Q$$

together with

$$P_1 = - P_0$$

gives

$$P_1 = 0.35 N_0 + 0.711Q$$

The other equations for the rivet loads are

$$P_2 = P_1 + 0.7 (N_0 + P_1) + 0.767Q = 1.2950 N_0 + 1.9757Q$$

$$P_3 = P_2 + 0.7 (N_0 + P_1 + P_2) + 0.067Q = 3.1465 N_0 + 3.9234Q$$

$$P_4 = P_3 + 0.7 (N_0 + P_1 + P_2 + P_3) - 0.433Q = 7.2006 N_0 + 8.1175Q$$

$$P_5 = P_4 + 0.7 (N_0 + P_1 + P_2 + P_3 + P_4) - 0.733Q = 16.2951 N_0 + 17.6938Q$$

and

$$P_6 = P_5 + 0.7 (N_0 + P_1 + P_2 + P_3 + P_4 + P_5) - 0.833Q = 36.7961 N_0 + 39.5558Q$$

which together with

$$P_6 = -P_5$$

give the maximum load in the plate and the rivet loads as

$$N_0 = -1.078Q \ (-2.382Q)$$

$$P_1 = +0.334Q \ (+1.286Q)$$

$$P_2 = +0.579Q \ (+1.000Q)$$

$$P_3 = +0.530Q \ (+0.715Q)$$

$$P_4 = +0.353Q \ (+0.429Q)$$

$$P_5 = +0.123Q \ (+0.143Q)$$

and

$$N_5 = +0.841Q \ (+1.191Q)$$

According to the classical theory (i.e., infinitely stiff rivets) the rivet loads are only dependent on the shear and are therefore the same for the continuous structure as for the structure with only a single span. These results are given above in brackets after the more accurate ones, and it is seen that the maximum rivet load and maximum load in the plate are only 45 percent of the values found by classical theory. Such great reductions can only be expected when the number of rivets is comparatively small and the rivets are not very stiff.

The above examples show that if the bending moment is changing sign, as in the case of a continuous structure, the plate takes a smaller proportion of the load than it does in the case of a simply supported structure. The slipping of the rivets reduces the load taken by the plate and the bending moment taken by the beam is correspondingly increased. According to classical theory, and for the dimensions assumed in the above examples, 51.2 percent of the total moment comes from the force N with arm a , while of the remainder, 48.2 percent comes from bending of the beam and 0.6 percent from bending of the plate. The more accurate value of N is, however, only 45.2 percent of the value given by classical theory, and hence only 23.2 percent of the total moment comes from N and 76.2 percent from bending of the beam. The maximum stress due to bending and axial forces in the beam thus increases by 32 percent (the bending moment increases but the axial force decreases).

The ratio of the load N to the value given by classical theory measures the extent to which the plate carries the bending moment. This ratio varies from 0.452 at the supports to 0.705 at midspan and the effective section of the structure is not therefore constant. The moment diagram assumed is only correct for constant stiffness, and if a more accurate solution to the problem is required a second order correction must be made. This correction does not affect the rivet loads and only introduces a change in the bending moment. It can easily be found by writing the total extension in the plate between two supports equal to zero. For example here

$$N = 0.011Q$$

and hence

$$N_0 = -1.067Q \quad \text{and} \quad N_5 = +0.952Q$$

Second Method - Rivets Replaced By A Continuous System Of Jointing

The rivets are not now treated as discrete members as in the solution given in First Method, but are replaced by a continuous system of jointing. The rivet loads P_1 are thus replaced by a continuously distributed load and the fundamental equations are found in the following way.

Let x be the distance from one end of the structure to the point considered. The axial load N is now assumed to be a continuous variable, and if l is the rivet pitch (now assumed to be constant) the rivet load is given by

$$P_1 = l dN/dx$$

and since

$$\delta_1 = CP_1 l / EA$$

it follows that

$$\delta = (Cl^2/EA) dN/dx$$

where the rivet slip δ is now also regarded as a continuous variable. The angle of bending ϕ_1 over the length l is replaced by the angle $d\phi$ over the length dx and it is then found that

$$M = E(I_1 + I_2) d\phi/dx + aN$$

The new equation formed by considering the extension at the common surface of the plate and the beam is

$$Ndx/EA_2 - t_2 d\phi/2 + \delta = -Ndx/EA_1 + (a - t_2/2)d\phi + \delta + d\delta$$

that is,

$$\begin{aligned} d\phi/dx &= \left\{ -d\delta/dx + N(1/A_1 + 1/A_2)/E \right\} /a \\ &= \left\{ - (Cl^2/A)d^2N/dx^2 + N(1/A_1 + 1/A_2) \right\} /Ea \end{aligned}$$

and substitution back into the above equation for M gives

$$M = (I_1 + I_2) \left\{ N(1/A_1 + 1/A_2) - (Cl^2/A_2)d^2N/dx^2 \right\} /a + aN$$

that is,

$$(Cl^2/A)d^2N/dx^2 - \left\{ 1/A_1 + 1/A_2 + a^2/(I_1 + I_2) \right\} N = -aN/(I_1 + I_2).$$

For simplicity write

$$k = (I_1 + I_2) \left\{ 1/A_1 + 1/A_2 + a^2/(I_1 + I_2) \right\} /a$$

$$b = \left[Cl^2/A \left\{ 1/A_1 + 1/A_2 + a^2/(I_1 + I_2) \right\} \right]^{\frac{1}{2}}$$

and

$$z = x/b$$

and then the differential equation for N takes the form

$$d^2N/dz^2 - N = M/k$$

The general solution of this equation is

$$N = Ae^z + Be^{-z} - e^z \int \left\{ e^{-2z} \int e^z (M/k) dz \right\} dz$$

where A and B are arbitrary constants to be determined from the fact that N is identically zero at each end of the structure.

Example 1

The above analysis will be applied to the same problem discussed in example 1 of First Method. It is found that for the same dimensions

$$k = 0.700 \text{ inch}$$

and

$$b = 2.390 \text{ inches}$$

It is most convenient to take the origin of coordinates at midspan and then

$$\begin{aligned} M &= (QL/8) \left\{ 1 - (2x/L)^2 \right\} \\ &= 2.5Q \left\{ 1 - (0.1x)^2 \right\} \\ &= 2.5Q \left\{ 1 - (0.1 \times 2.390z)^2 \right\} \\ &= 2.5Q (1 - 0.05714z^2) \end{aligned}$$

and $z = 0$ at midspan and $z = \pm 1/0.2390 = \pm 4.184$ at the ends of the structure. From the value of M it is then found that

$$N = Ae^z + Be^{-z} + Q(3.163 - 0.204z^2)$$

Now there is symmetry about the origin and therefore A is equal to B. Further N is zero for $z = 4.184$ and this finally gives

$$A = B = 0.0062Q$$

At midspan, that is for $z = 0$,

$$N = (2 \times 0.0062 + 3.163)Q = 3.175Q$$

and this result is in good agreement with the value of 3.172Q found previously.

The rivet loads may now be found from the change in the value of N over intervals corresponding to the rivet pitch. If, however, the number of rivets is not large, the method given in First Method is simpler for practical use.

SOLUTION APPLICABLE TO LOADS BEYOND THE PROPORTIONAL LIMIT FOR THE RIVETS AND THE PLATE, BUT NOT FOR THE BEAM - RIVETS TREATED AS DISCRETE MEMBERS

In order to simplify the analysis of the problem the beam is assumed not to be stressed beyond the limit of proportionality. Nonlinear deformations are, however, assumed to occur both in the rivets and in the plate. The cross-sectional area of the structure is assumed to be constant and the rivet pitch and stiffness are assumed to be the same for all the rivets.

Above the limit of proportionality the equations that determine the load distribution are no longer linear and, although an exact solution may be formally obtained by treating δ as a nonlinear function of P , the computational work would then be very severe. The results may, however, be obtained to any required degree of accuracy in the following simple way, provided the load-extension curve is known.¹ Assume that the load on the i^{th} rivet is P_i and then near this value

$$\delta_i = k_i (P_i - S_i) \delta_0$$

where the meaning of the constants k_i and S_i may be seen from figure 7. The quantity k is proportional to the reciprocal of the tangent modulus in the same way that C is proportional to the reciprocal of the modulus of elasticity (E) at low loads. The stress f in the plate may also be approximately assumed to be a linear function of the strain ϵ within a certain range of nonlinear deformations, that is

$$f = f_0 + K\epsilon$$

where K is the tangent modulus and f_0 is a constant stress.

Now let

$$A_2^{-1} = KA_2/E \quad \text{and} \quad I_2^{-1} = KI_2/E$$

and then the bending moment necessary to produce an angle of bending ϕ over the length l is

¹This method is also used in reference 1.

$$M_2 = \int f y dA = \int (f_0 + K\phi y/l) y dA = K\phi I_2/l = E\phi I_2^1/l$$

where y is the distance from the middle surface of the plate and dA is an element of area of the plate.

Then the total bending moment in the structure is

$$M = E(I_1 + I_2^1)\phi/l + aN_f$$

The elongation ϵ of the section l under the axial load $N_1 = F A_2$ is given by

$$\epsilon l = (f - f_0)l/K = (N_1 - T)/KA_2 = (N_1 - T)/EA_2^1$$

where

$$T = f_0 A_2$$

The equation connecting the slip of successive rivets is now found to be

$$(N_1 - T)l/EA_2^1 - t_2\phi/2 + \delta_1 = -N_1l/EA_1 + (a - t_2/2)\phi + \delta_{1+1}$$

and it then follows that

$$P_{1+1} = S_{1+1} + (k_1/k_{1+1}) (P_1 - S_1) + (A_0/k_{1+1})$$

$$\times \left[\left\{ 1/A_1 + 1/A_2^1 + a^2/(I_1 + I_2) \right\} N_1 - T/A_2^1 - aM_1/(I_1 + I_2^1) \right]$$

In most cases it is probably accurate enough to assume that

$$k = k_1 = k_{1+1} \quad \text{and} \quad S_1 = S_{1+1}$$

for all rivets loaded above the limit of proportionality, and then

$$P_{1+1} = P_1 + (A_0/k) \left[\left\{ 1/A_1 + 1/A_2^1 + a^2/(I_1 + I_2^1) \right\} N_1 - T/A_2^1 - aM_1/(I_1 + I_2^1) \right]$$

It must be remembered, however, that even if only one rivet is loaded within the limit of proportionality and the adjacent rivets above this limit, the complete formula must be used. For example, if

$$\delta_1 = k_1 (P_1 - S_1) \delta_0$$

and

$$\delta_{i+1} = CP_{i+1} \delta_0$$

then

$$P_{i+1} = (k_1/C)(P_1 - S_1) + (A_0/C) \left[\left\{ 1/A_1 + 1/A_2^1 + a^2/(I_1 + I_2^1) \right\} N_1 - T/A_2^1 - aM_1/(I_1 + I_2^1) \right]$$

Example 1

The structure discussed in example 1 of First Method is now assumed to be loaded above the limit of proportionality.

For the rivets it is assumed that

$$k = 4C \quad \text{and} \quad S = 0.15 \text{ ton}$$

thus giving nonlinear deformations for loads above

$$kS/(k - C) = 0.20 \text{ ton}$$

which corresponds to an average shear stress of 10.5 tons per square inch. For the plate it is assumed that

$$k = E/4 \quad \text{and} \quad T = 1.35 \text{ tons}$$

thus giving nonlinear deformations for loads above

$$ET/(E - k) = 1.8 \text{ tons}$$

which corresponds to an average stress of 15.5 tons per square inch.

It has already been found that for loads below the limit of proportionality

$$P_1 = 1.059Q$$

$$P_2 = 0.900Q$$

$$P_3 = 0.671Q$$

$$P_4 = 0.412Q$$

and

$$P_5 = 0.141Q$$

The proportional limit is therefore reached when $Q = 0.189$ ton (this gives $P_1 = 0.2$ ton) while the load in the plate is still far below this limit.

If the load is increased nonlinear deformations occur in the first rivet and suppose that the increase is just up to the limit of proportionality for the second rivet. The rivet slip for the first and second rivets may be written as

$$\delta_1 = k(P_1 - S_1) \delta_0$$

and assuming that $k = 4C$, this gives

$$P_2 = P_1 + (1/4) (0.7 P_1 - M_1) = 1.175 P_1 - 0.25 M_1$$

The rivet slip for the second and third rivets may be written as

$$\delta_1 = CP_1 \delta_0$$

and then as before

$$P_3 = P_2 + 0.7 (P_1 + P_2) - M_2$$

Similarly, the other equations are the same as those previously obtained for loads within the limit of proportionality. Hence

$$\begin{array}{ll}
 P_1 = 0.990Q & N_1 = 0.990Q \\
 P_2 = 0.938Q & N_2 = 1.928Q \\
 P_3 = 0.687Q & N_3 = 2.615Q \\
 P_4 = 0.418Q & N_4 = 3.033Q \\
 P_5 = 0.141Q & N_5 = 3.174Q
 \end{array}$$

and these values are valid up to $Q = 0.213$ ton (this gives $P_2 = 0.2$ ton).

The load is now increased up to the limit of proportionality for the third rivet. The equations to be solved are then

$$P_2 = P_1 + (1/4) (0.7 P_1 - M_1)$$

$$P_3 = P_2 + (1/4) \left\{ 0.7 (P_1 + P_2) - M_2 \right\}$$

$$P_4 = P_3 + 0.7 (P_1 + P_2 + P_3) - M_3, \text{ and so forth.}$$

which gives $P_3 = 0.2$ ton for $Q = 0.260$ ton. In this way the rivet loads may be successively found for $Q = 0.358$ ton giving $P_4 = 0.2$ ton and for $Q = 0.741$ ton giving $P_5 = 0.2$ ton. The numerical results for the five cases are given in the table below:

Q (tons)	0.189	0.213	0.260	0.358	0.741
P_1 (tons)	<u>0.200</u>	0.211	0.240	0.308	0.595
P_2 (tons)	0.170	<u>0.200</u>	0.223	0.282	0.532
P_3 (tons)	0.127	0.147	<u>0.200</u>	0.242	0.432
P_4 (tons)	0.078	0.089	0.117	<u>0.200</u>	0.317
P_5 (tons)	0.027	0.030	0.039	0.064	<u>0.200</u>
N_5 (tons)	0.602	0.678	0.819	1.096	2.076

Between these values the rivet loads will vary linearly with the total load Q .

The ultimate strength of the first rivet is, however, likely to be reached before the fifth rivet is loaded to the limit of proportionality, and further the plate load N_5 is also likely to have already exceeded the limit of proportionality. The tabulated values are therefore probably only valid up to some value of Q between 0.358 ton and 0.741 ton, and if for example the ultimate strength of the rivets is 0.35 ton, this value is 0.414 ton giving $P_1 = 0.35$ ton and $N_5 = 1.29$ tons. Classical theory gives $P_1 = 0.533$ ton and $N_5 = 1.48$ tons, and it follows that rivet slip has reduced these loads by 34 percent and 16 percent, respectively.

Example 2

It has been shown that with the same dimensions as in example 1 of First Method the ultimate strength of the rivets is reached before the loads in the plate exceed the limit of proportionality. In order to make the rivets and the plate carry loads above this limit at the same time, the span of the structure may be increased while the cross section and riveting remain unchanged. The structure considered in this example is shown diagrammatically in figure 8. The distributed load is now replaced by a concentrated load (Q); the rivet pitch is unaltered, but rivets are now situated immediately over the supports.

The vertical shear load now has the constant value of $Q/2$, and according to classical theory the rivet loads are given by

$$P_1 = 0.7143Q$$

and

$$P_2 = P_3 = \dots = 1.4286Q$$

The bending moments at sections midway between the rivets are

$$M_1 = 0.5Q$$

$$M_2 = 1.5Q$$

$$M_3 = 2.5Q$$

$$M_4 = 3.5Q$$

$$M_5 = 4.5Q$$

$$M_6 = 5.5Q$$

and

$$M_7 = 6.5Q$$

and the maximum bending moment at mid-span is $M = 7Q$, which according to the classical theory gives a maximum load in the plate of $N = 10Q$. For loads below the limit of proportionality the equations to be solved are

$$P_2 = P_1 + 0.7 P_1 - 0.5Q$$

$$P_3 = P_2 + 0.7 (P_1 + P_2) - 1.5Q$$

$$P_4 = P_3 + 0.7 (P_1 + P_2 + P_3) - 2.5Q$$

.....

$$P_8 = P_7 + 0.7 (P_1 + P_2 + \dots + P_7) - 6.5Q$$

From symmetry

$$P_8 = 0$$

and then

$$P_1 = 1.025Q$$

$$N_7 = 8.267Q$$

$$P_2 = 1.242Q$$

$$P_3 = 1.328Q$$

$$P_4 = 1.344Q$$

$$P_5 = 1.301Q$$

$$P_6 = 1.170Q$$

$$P_7 = 0.857Q$$

$$P_8 = 0$$

Because the number of rivets is rather large, the maximum rivet load is found to be as much as 94 percent of the value given by classical theory. Not all the rivets, however, reach so high a percentage, and the load in the plate is therefore only 82.7 percent of the classical value. If non-linear deformations occur for rivet loads above 0.2 ton, the above results are only valid up to $Q = 0.179$ ton (this gives $P_4 = 0.2$ ton).

Suppose now that the load is increased until nonlinear deformations occur in all rivets except the eighth which, from symmetry, still carries no load. As before, the value $k = 4C$ is taken and the equations to be solved are

$$P_2 = P_1 + (1/4) (0.7 P_1 - 0.5Q)$$

$$P_3 = P_2 + (1/4) \left\{ 0.7 (P_1 + P_2) - 1.5Q \right\}, \text{ and so forth.}$$

If the seventh rivet is just loaded to the limit of proportionality (0.2 ton)

$$0 = P_8 = P_7 + 0.7 (P_1 + \dots + P_7) - 6.5Q$$

and beyond this limit

$$0 = P_8 = (4C/C) (P_7 - S) + 0.7 (P_1 + \dots + P_7) - 6.5Q$$

that is,

$$P_7 - 0.15 + 0.25 \left\{ 0.7 (P_1 + \dots + P_7) - 6.5Q \right\} = 0$$

From this last equation

$$P_1 = 1.0524Q + 0.0136$$

$$P_2 = 1.1116Q + 0.0160$$

$$P_3 = 1.1152Q + 0.0211$$

$$P_4 = 1.0641Q + 0.0300$$

$$P_5 = 0.9492Q + 0.0441$$

$$P_6 = 0.4504Q + 0.0659$$

$$P_7 = 0.4830Q + 0.0992$$

$$P_8 = 0$$

$$N_7 = 6.5259Q + 0.2899$$

and these results are valid from $Q = 0.2087$ ton, giving $P_7 = 0.2$ ton, to $Q = 0.2314$ ton, giving $N_7 = 1.8$ tons, which is the assumed limit of proportionality for the plate.

Beyond this load nonlinear deformations also occur in the plate at the seventh section and, introducing the quantities A_2^1 and I_2^1 , the equation for P_8 is then

$$0 = P_8 = 4(P_7 - S) + 1.472 (P_1 + \dots + P_7) - 1.042T - 1.01 \times 6.5Q$$

that is,

$$P_7 - 0.15 + 0.25 \left\{ 1.483(P_1 + \dots + P_7) - 1.042 \times 1.35 - 1.01 \times 6.5Q \right\} = 0$$

This gives

$$P_1 = 0.9677Q + 0.0329$$

$$P_2 = 1.0120Q + 0.0386$$

$$P_3 = 0.9835Q + 0.0512$$

$$P_4 = 0.8771Q + 0.0727$$

$$P_5 = 0.6741Q + 0.1070$$

$$P_6 = 0.3392Q + 0.1600$$

and

$$P_7 = 0.1365Q + 0.2409$$

and these values are valid up to $Q = 0.276$ ton, when nonlinear deformation in the sixth section of the plate commences. It is then necessary to use the following equation for P_7 ,

$$P_7 = P_8 + 0.25 \left\{ 1.483 (P_1 \dots + P_6) - 1.042 \times 1.35 - 1.01 \times 6.5Q \right\}$$

and the other equations are as given above. Then

$$P_1 = 0.9053Q + 0.0518$$

$$P_2 = 0.9387Q + 0.0609$$

$$P_3 = 0.8863Q + 0.0805$$

$$P_4 = 0.7392Q + 0.1143$$

$$P_5 = 0.4715Q + 0.1682$$

$$P_6 = 0.0461Q + 0.2514$$

and

$$P_7 = 0.1217Q + 0.1693$$

and these values are valid up to $Q = 0.336$ ton when nonlinear deformation in the fifth section of the plate commences, provided the first rivet has not already reached its ultimate strength. Similarly, results may be found as nonlinear deformations spread to other sections of the plate, and between these characteristic values of Q the rivet loads are linearly dependent on the total load Q . It is seen that when there are many rivets the maximum rivet load does not differ greatly from the value given by the classical theory so long as the loads do not exceed the proportional limit. There is, however, a considerable difference when the loads are above this limit. For this particular example the maximum rivet load at the ultimate load is only about 78 percent of the classical value. The plate load is always comparatively less, and again for this case is only 68 percent of the classical value.

Numerical results have also been obtained when the rivets are assumed to be rigid and very closely pitched while the other dimensions and the loading are the same as those above. A comparison with the previous results shows that for loads near the ultimate strength of the structure, the major portion of the reduction in the load in the plate is due to nonlinear deformation of the plate, and that rivet slip only contributes to this reduction to a lesser degree.

CONCLUSIONS

The examples given show that the loads carried by the rivets and the plate are less than the values given by the classical theory, which does not take into account the slip of the rivets under load, even when the loads are within the limit of proportionality. This difference is considerably accentuated when nonlinear deformations occur in the structure, and the beam then carries the greater portion of the bending moment. If the material of the beam has a higher proportional limit and a higher ultimate strength than the material of the plate, there is a transfer of load from weaker to stronger material and this is to the advantage of the structure. The effect of the slipping of the rivets is not only dependent on the characteristics of each rivet but also on the number of the rivets and the variation of the bending moment along the structure. The examples also show that if there is only a small number of rivets in a section of the plate with maximum tension and maximum compression (as for continuous beams) the load in the plate is considerably less than that given by classical theory. Further, even if the number of rivets is large, the rivets and the plate evade a very considerable part of this load if they are stressed beyond the limit of proportionality.

The methods described are of simple application and are recommended for use in the design of light-alloy structures when the design load is likely to be above the proportional limit.

REFERENCE

1. Vogt, F.: The Load Distribution in Bolted or Riveted Joints in Light-Alloy Structures. NACA TM No. 1135 (to be published).

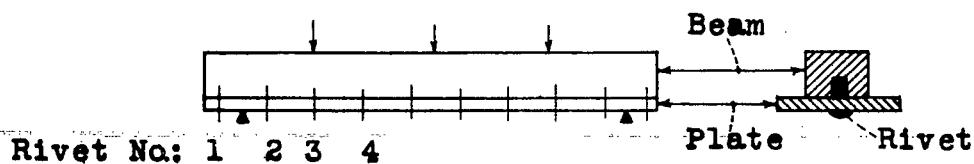


Figure 1.

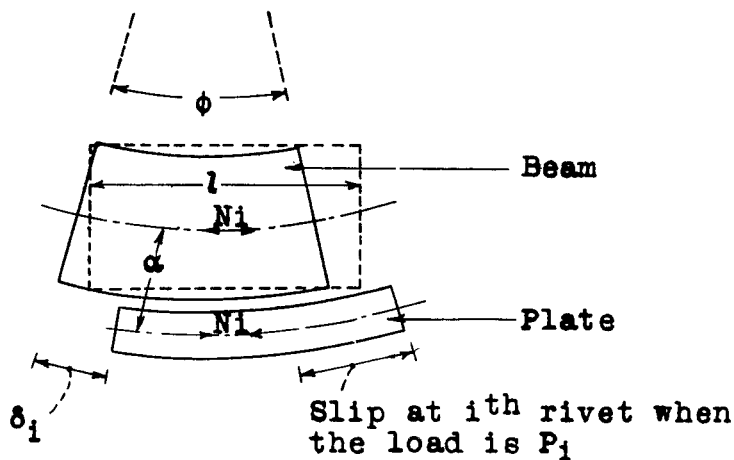


Figure 2.

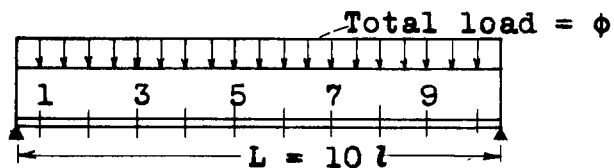


Figure 3.

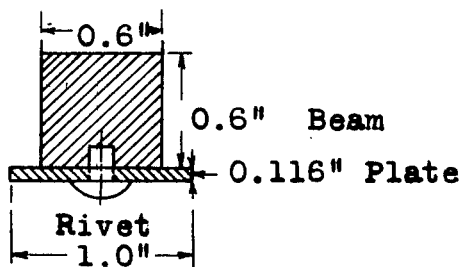


Figure 4.

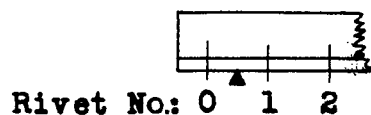


Figure 5.

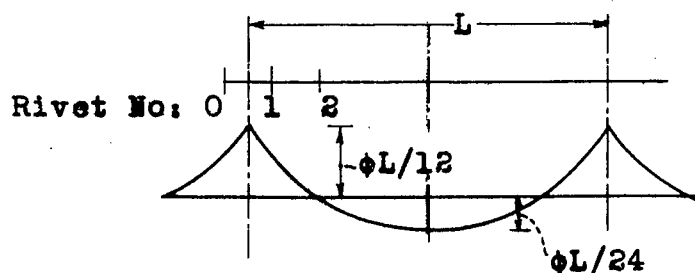


Figure 6.- Variation of bending moment along the span for a continuous beam under uniform transverse load. Total load = ϕ .

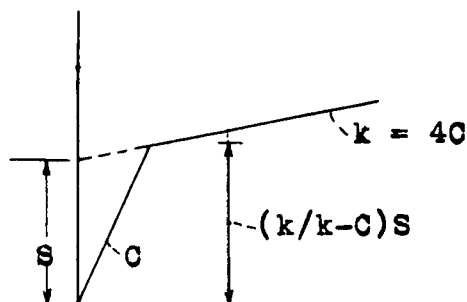


Figure 7.- Representation of load-extension curve by two straight lines.

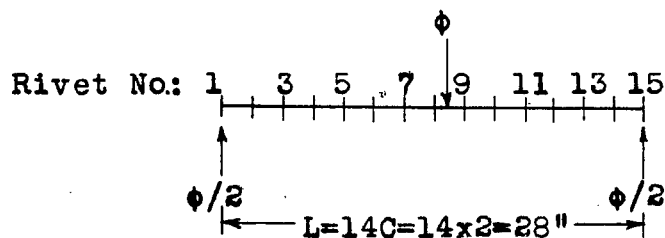


Figure 8.

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